

FINITE ELEMENT METHOD FOR SOLVING ORDINARY & PARTIAL DIFFERENTIAL EQUATIONS

MATH 4400/8406

Course Description:

Introduction to finite elements methods for solving ordinary and partial differential equations. Theoretical foundations of finite element methods for boundary value problems, approximation by piecewise polynomial functions, variation formulation of partial differential equations, basic error estimates. The Rayleigh-Ritz-Galerkin method, convergence of approximations, time-dependent problems, error analysis, discretization and computer implementation, applications to problems in elasticity, heat transfer, and fluid mechanics. **3 credits**

Prerequisites:

Math 1970 with a C- or better, Math 2050 with a C- or better, and Math 2350 with a C- or better, or instructor's permission. MATH 3300/8305 and MATH 4330/8336 are recommended, but not required. Familiarity with MATLAB programming is assumed.

Overview of Content and Purpose of the Course:

The finite element method is a numerical technique of solving differential equations arising in the analytical description of physical phenomena, such as heat transfer, fluid mechanics, and solid mechanics. This course is an introduction to the finite element method to solve ordinary and partial differential equations. It will cover the numerical and theoretical foundation of finite element methods for differential equations. The course covers the basic topics of finite element technology, including domain discretization, polynomial interpolation, application of boundary conditions, assembly of global arrays, and solution of the resulting algebraic systems. Emphasis will be on one- two- and three-dimensional problems. The course will also cover the Galerkin method for one-, two- and three-dimensional differential equations (including parabolic, hyperbolic and elliptic partial differential equations). Convergence and stability criteria are developed. Extensive time will be devoted to computational aspects, with only limited time on theoretical analysis, including a review of functional analysis. Discontinuous Galerkin methods, adaptive refinement strategies, and error estimation techniques for typical finite element methods will also be discussed.

Anticipated audience/demand:

. This course is designed for graduates and undergraduates majoring in Mathematics and Engineering needing a basic familiarity with techniques for solving differential equations by the finite element method.

Major Topics:

Lectures will emphasize foundational concepts in applied mathematics and the processes by which we gain new knowledge. Major Lecture Topics:

1. An Introduction to the Use of Finite Element Procedures (Physical Problems and Mathematical Models; Applications of Finite Element Methods in Engineering and Physics).
2. Mathematical Preliminaries (Set Notation, Function Notation, Vectors, Matrices, Tensors, Differential Equations, Partial Differential Equations, Variational Calculus).
3. Introduction to FEM using One-Dimensional Problems.
4. Variational Principles and Methods of Approximations.
5. Finite Element Basis Functions
 - a. Two Dimensional Elements (Rectangular Elements and Triangular Elements)
 - b. Three Dimensional Elements (Hexahedral Elements and Tetrahedral Elements)
6. Computational Aspects
 - a. Mesh Generation
 - b. Numerical Integration
 - c. Assembly
 - d. Algebraic Systems
7. Error and Convergence Analysis
 - a. A Priori Estimates
 - b. A Posteriori Estimates
8. Parabolic Problems
9. Hyperbolic problems
 - a. Standard Finite Element Method
 - b. Discontinuous Finite Element Method
10. Adaptive Methods

Methods:

The class will be presented primarily in lecture form with student discussion encouraged. Questions are encouraged in class and out. We will use MATLAB as a tool for analyzing problems. Examinations may also be used to help students learn and retain important objective material relating to the topic.

Student Role:

Students must attend lectures, participate in discussions, and complete the written homework and programming assignments.

Textbook:

Johnson, Claes. *Solution of Partial Differential Equations by the Finite Element Method*. Mineola: Dover Publications, 2009.

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